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DETECTION PROBABILITY COMPUTATIONS FOR RANDOM SEARCH OF AN EXPANDING AREA

by

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[redacted]

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PREFACE

The discussion in this paper of an approach to the problem of locating a target in an area of uncertainty grew out of an investigation* of the factors limiting the use of submarine sonar at high speeds. This report has been prepared to assist the Committee on Undersea Warfare in clarifying questions that have arisen relative to the search rate of submarines in comparison to other ASW platforms. This subject is recognized as being of interest to a rather wide audience. Consequently, the report is being distributed to the Committee and, in addition, to those who, it is felt, should find it useful. The calculator included with the report should have particular application in the planning and evaluation of operational ASW exercises as well as the analysis of the potential performance of searching systems.

Information on passive sonar search rates was found to be not generally available even though it was felt to be an important consideration in ASW tactics, analysis, and planning. It is hoped that this report will serve to stimulate further inquiry into the subject of search rates of competitive platforms and consideration of the choice of the most appropriate vehicle in tactical situations.

This treatment of detection probability computations has purposely been made as broad as possible in order that the methods may be tried on a wide range of search situations. For this reason and also to ensure that it is completely unclassified, the calculator scales have been extended well above and below the parameters normally encountered in ASW practice.

* *A Survey of Submarine Technology, Volume II: Submarine Search Rate (U)* by Tetsuo Arase and Elizabeth M. Arase. December 1970. NRC:CUW:0373. [CONFIDENTIAL]

CONTENTS

INTRODUCTION

Search theory is a well-developed branch of military operations research; the wartime work of Koopman and colleagues is summarized in the declassified report, "Search and Screening," (1946).* Postwar reports on search theory and applications appear in the journals *Operations Research* and *Naval Research Logistics Quarterly*, the publications of the various military operations research groups, and elsewhere.

The purpose of this brief paper is to apply basic search theory to a particular problem in random search - search about a "datum," or "point of fix" - and to derive computationally convenient formulas and computational aids for detection probability in this situation, noting the assumptions and approximations underlying the use of these formulas. With these ends in view, the mathematical development is presented without detailed references, but the reader unfamiliar with the basic work in this field may wish to refer to the literature noted above.

The situations in which the techniques described later in this paper might apply are as follows. A long-range detection system with a cross-fix capability has detected a target but has not been able to establish a reliable target track. The system has a bearing error which, at the long distances involved, results in a sizable area within which the exact location of the target is unknown. Figure 1 illustrates this.

* Report No. 56 of the Operations Evaluation Group, Office of the Chief of Naval Operations; also issued as Volume 2B, Division 6 in the series of Summary Technical Reports of the National Defense Research Committee.

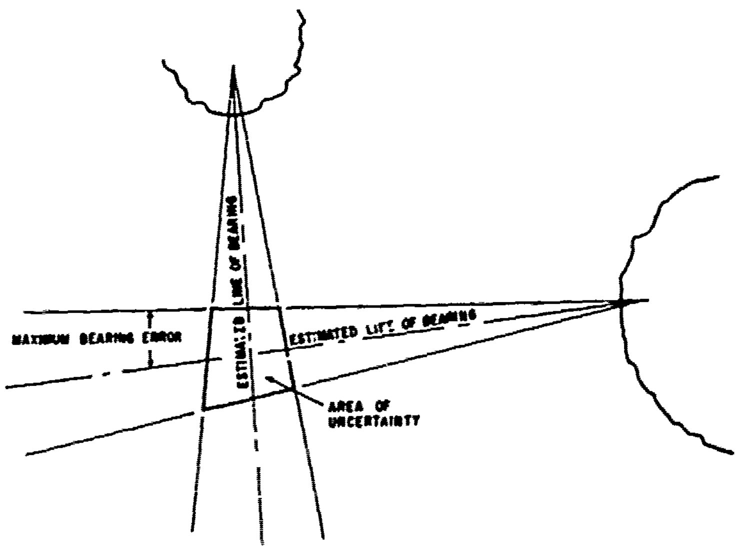


Figure 1

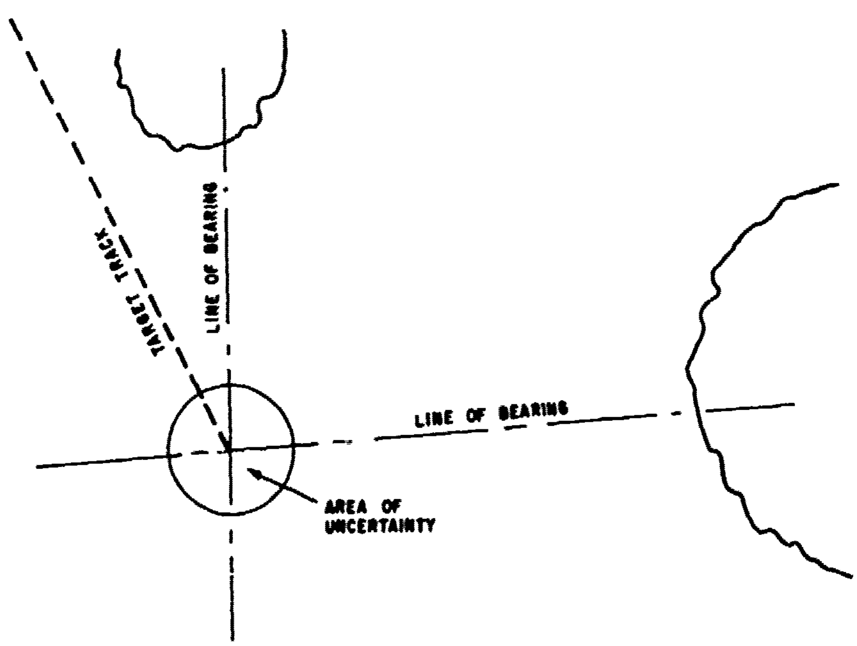


Figure 2

Another version of this situation is where a detection system with good bearing accuracy has detected and held a target long enough to establish its track, but then course changes are made, track information deteriorates, and at a certain point the target is lost. This is shown in Figure 2.

A third alternative version that might arise is where one vehicle is following another with the objective of keeping the latter under surveillance. The trailer has detected and trailed the target and established a course and speed but then the contact is lost and the target is presumed to have made arbitrary changes of course and/or speed. This is illustrated in Figure 3.

In all of these cases no additional information about the exact location of the target is forthcoming. Because the target must be assumed to have mobility, the area of uncertainty increases rapidly with time. Figure 4 graphically depicts the growth of an uncertainty area. The area of uncertainty, which is shown on the ordinate, can be expressed in square miles; time since loss of contact, shown on the abscissa, is in hours.

When several vehicles are available which might localize the target within the area of uncertainty, a basis is needed for deciding which would find the target quickest.

When the candidate searching vehicles are already within the area to be searched, the assignment should be given to the vehicle with the greatest search rate. This point is illustrated by Figure 5 wherein it can be seen that a high-search-rate vehicle, labeled A, will probably locate the target in the shortest time, t_1. A vehicle, B, with a rate just sufficient on the average to locate the target will take a longer time, t_2. A low-search-rate vehicle, C, will probably never find it.

A second level of decision arises when the search vehicles do not happen to be within the uncertainty area and must transit varying distances to initiate the search. It is quite possible under these circumstances that the vehicle with a lower search rate, if it were near the search area, would have the greater probability of locating the target. This is shown in Figure 6 where vehicle B with a shorter transit time, t_1, to the search area, locates the target sooner than vehicle A, arriving in the area later (at t_2).

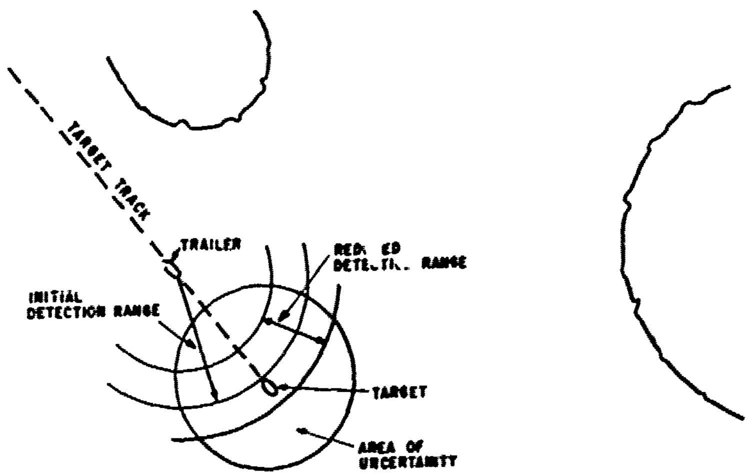


Figure 3

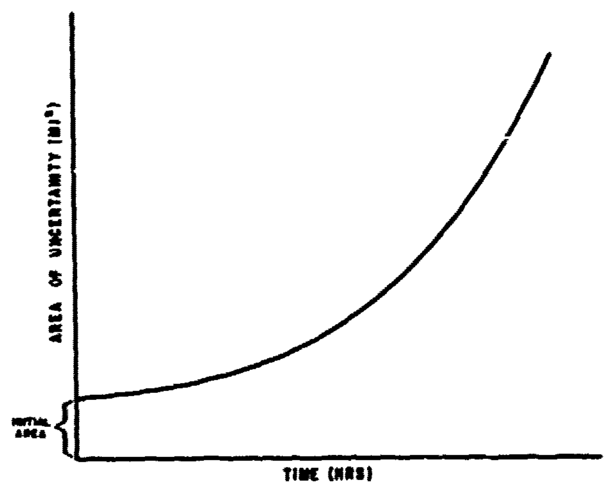


Figure 4

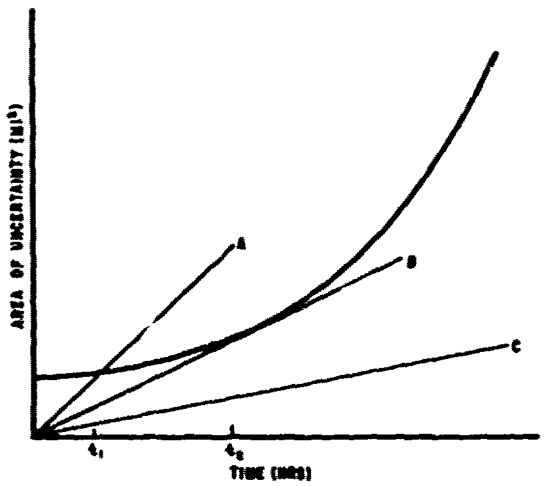


Figure 5

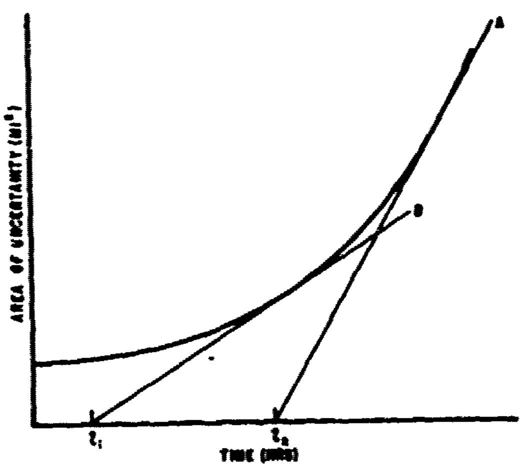


Figure 6

The possibility of employing vehicles with markedly different search rates in a team effort to locate a target should also be considered. This might arise where the expense of maintaining high-search-rate vehicles with reasonable transit time of all possible areas of uncertainty would be too great. But on the other hand, it might be economically feasible to preposition a number of lower-search-rate vehicles so that their transit time to the area would be relatively short.

THE CONCEPT OF SEARCH RATE

The performance of a search vehicle (an aircraft, ship, or submarine) looking for a target or targets in some well-defined area, may be characterized by a search rate, S (usually measured in square miles per hour) for targets of a particular type. This rate is essentially an average value of the product of the area searched per unit time and the fraction of targets within that area that are detected. Repeated trials under similar conditions serve to establish empirical estimates of S for existing detection systems and target types; extrapolation to other types of detection systems and targets is often made by application of detection range equations for sonar, radar, or visual search. However derived, the search rate S is central to the use of formulas for random-search detection probability.

Although the development that follows takes the search rate for a particular search vehicle as given, it should be appreciated that the quantity varies, not necessarily linearly, with the searching speed. At low speeds the variation may be linear, but as speed increases an optimum is reached above which further speed increases will actually yield decreased search rates.

RANDOM SEARCH ABOUT A POINT OF FIX

Suppose that at some particular time prior to the start of a search, a target is known simply to be somewhere in the vicinity of a specified point (the "datum" or "point of fix"). For conceptual and computational simplicity, we consider the target as localized within a circle whose radius is increasing at a rate equal to the assumed speed of the target, whose direction of movement is assumed to be unknown and freely changeable.

(Other shapes of the localization region, such as quadrilaterals or ellipses, tend to become more nearly circular with the passage of time from the latest pre-search datum and can often be reasonably approximated by circles of equivalent area.)

At the time the search vehicle arrives in the vicinity of the target and begins search, we assume the target to be somewhere within a circle of fixed radius R. After t hours of fruitless search, a target of (approximately) known speed U, but of unknown and randomly chosen direction, is assumed equally likely to be anywhere within a circle of radius $R + Ut$. Note that this is an *assumption* whose plausibility should be examined for the particular search situation considered. The assumption is rarely if ever precisely satisfied, but it is often a fair approximation in diverse search situations; uncertainties in the input parameter values will likely outweigh such error as may be introduced by this assumption of uniform probability of location over the expanding localization circle. There is an important class of search situations, however, in which this assumption fails badly, and the formulas of random search should not be used without radical modification: this is the situation, often arising in *active* sonar or radar search, in which a hostile target's detection ability outranges the searcher's and enables him to make effective evasive maneuvers. In such situations, the target is distinctly *not* equally likely to be anywhere within the localization circle; he may choose his speed and direction of travel purposefully to keep out of the searcher's detection range. Similar modifications may be required in certain search and rescue operations, where a cooperative target may maneuver so as to enhance his probability of detection.

In those search situations, then, in which the foregoing assumptions are deemed reasonable approximations, we have an expanding localization circle whose area, $A(t)$, after search time, t, is

$$A(t) = \pi(R + Ut)^2, \qquad (1)$$

and this expanding area is searched at a presumed constant search rate, S. Thus, in any brief period, dt, the fraction of the circle swept out (and the incremental probability, dP, of detecting a previously undetected target) is

$$dP = S\,dt/A(t) \tag{2}$$

The "coverage factor," C, may be taken as the effective fraction or multiple of the localization circle's expanding area which has been covered after a search time, T; it is computed by integration of formula (2):

$$\begin{aligned} C &= \int_0^T S/A(t)\,dt \\ &= (S/\pi)\int_0^T (R + Ut)^{-2}\,dt \\ &= [S/(\pi RU)]\;[UT/(R + UT)]. \end{aligned} \tag{3}$$

Equation (3) lends itself to rapid calculation, given the four input parameters, R, S, T, and U. From the coverage factor, C, the detection probability, P, may be calculated, according to the formula for random search*:

$$P = 1-\exp(-C). \tag{4}$$

In explanation of Eq. (4), it should be noted that random search entails random gaps and overlaps in coverage, so that the localization circle can never be swept entirely "clean." When, for instance, the circle has been covered just once, i.e., when $C = 1$, the detection probability under the random-search assumption is substantially less than unity; in this case it is

$$P = 1-\exp(-1)=1-1/2.71828...=0.63,$$

and double coverage, $C=2$, raises P just to

$$P = 1-\exp(-2)=0.86$$

Equation (4) may be simply derived, and better understood, by considering the search as consisting of a large number, n, of brief independent glimpses, each of which gives one nth of the total coverage C. The probability, q, of *non*detection in each "glimpse" is

$$q = 1-(C/n),$$

and the probability, Q, of nondetection in all n, glimpses is, by multiplication of probabilities of independent events

* A brief derivation and discussion of the formula for random search appears on pages 28 and 29 of the OEG/NDRC report, "Search and Screening" cited earlier.

$$Q = q^n = [1-(C/n)]^n.$$

Thus the resultant detection probability, P, is

$$P = 1-Q = 1-[1-(C/n)]^n \quad (3)$$

and, in the limit as n approaches infinity, the term

$$[1-(C/n)]^n$$

approaches exp $(-C)$. Thus

$$P = 1-\exp(-C). \quad (4)$$

A NOTE ON "RANDOM" VERSUS "IDEAL" SEARCH

The phrase "random search" requires care in interpretation. In the previous sections we have used the "formula for random search," formula (4), to establish detection probability resulting from a search in which the "coverage factor" is computed to be C. The reader may well inquire whether, with a carefully planned systematic search, detection probability might not be better estimated by the value C (up to the point where C reaches unity) rather than the smaller value $P=1-\exp(-C)$. Indeed, under certain ideal conditions, detection probability closely approximates the value C, even as C becomes nearly 1. The ideal conditions required, however, are rather implausible in most practical search operations: (1) The detection system must "sweep clean" in the sense that it detects every target within its presumed detection range and none outside, (2) the search must be so planned that there is no overlap in the areas swept out, and (3) the target must not be able to double back and find sanctuary in a previously swept area. These ideal conditions run counter to the presumed freedom of motion of the target and the widely observed variability of actual detection ranges from the presumed average. The question, then, is whether the detection probability P expected in a particular search situation is better estimated by the formula of "random search",

$$P = 1-\exp(-C), \quad (4)$$

or by a formula of "ideal search",

$$P = C \text{ for } C < 1,\ P = 1 \text{ for } C \geq 1. \tag{6}$$

More simply written,

$$P = \min(C,1). \tag{6'}$$

As shown in Figure 7, the formula for random search, (4), always gives a smaller value for P than the "ideal search formula" (6), but the greatest difference, about 37%, occurs when $C = 1$. For C much smaller or larger than 1, the discrepancy is much less (for example, for $C = 0.5$, (4) gives a value P of 0.395, and for $C = 2$, (4) gives P = 0.865) and is seldom worth worrying about.

The more important concern is to plan the search so that gaps and overlays in coverage are no *worse* than assumed in the formula for random search. Indeed, "random search" does *not* mean unplanned or disorderly search; random search usually requires careful planning and systematic execution.

SIMPLIFIED DETECTION PROBABILITY COMPUTATIONS

Detection probability for various combinations of the parameters R, U, S, and T may readily be calculated by slide rule, tables, or digital computer using Eqs. (3) and (4). It is convenient, however, for repeated use of these equations, to make use of special computational aids: graphs, nomographs, or a special-purpose probability slide rule based on Eqs. (3) and (4). All three forms of computational aid have been prepared, in prototype form, for a suitable range of parameter values of interest in practical search problems. The special-purpose slide rule has been found to be the most convenient for practical use, and a slide rule has been included in the folder at the end of this report, together with instructions for its use.

For purposes of nomographic and specialized slide rule computation it was necessary to make a change of variable in Eq. (3), defining a "head start" time, H, as follows:

$$H = R/U.$$

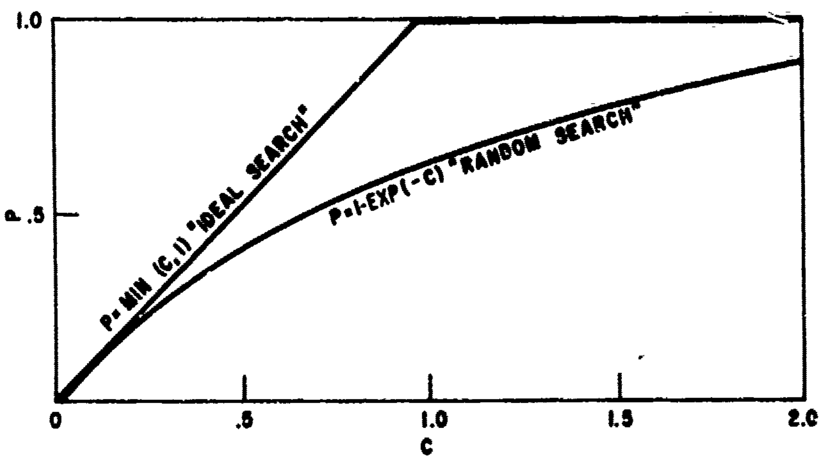


Figure 7

The quantity, $H=R/U$ may be regarded as the effective head start the target has on the searcher. This head start is not quite the same as searcher time late after datum, except in the special case where a precise fix has been made on the target initially. Thus, the notion of head start embodies both the initial localization uncertainty (a 5-knot target initially localized within a circle of 10-mile radius being regarded as having effectively an initial 2 hr. head start) and the added searcher time late.

In place of Eq. (3) giving coverage factor, C, as a function of T, we now have a somewhat simpler form giving coverage factor, C, as a function of search time, T, and head start time, H.

$$C(T,H) = (S/\pi RU)\ (T/(H+T)) \tag{3'}$$

from which we may readily compute

$$P\ (T,\ H) = 1\text{-exp}\ [-C(T,H)]. \tag{4'}$$

Detection probability computations for random search about a point of fix can thus be readily made by a special-purpose circular slide rule, based on Eqs. (3') and (4').

Properly applied, and with the underlying assumptions of the random-search formulas kept clearly in mind, these computations may be useful both tactically, in search planning and execution, and in systems comparisons. Careful judgment must, of course, be used· our present assumptions normally do *not* hold (1) in most *active* search situations, (2) when the localization region starts and remains substantially noncircular (say, a quadrilateral of grossly unequal diagonals or a very elongated ellipse), (3) when direction of target travel is restrained by mission or geography, or approximately known, or (4) when the basic surveillance system (or other source) provided refined or updated localization on the target during the local search. Adaptations of the basic random-search concept may be made in certain of these cases, but they go beyond the scope of the present discussion.

In conclusion, two interesting features of the coverage factor, Eqs. (3) and (3'), should be noted. First, the term $S/\pi RU$ represents an upper limit to search coverage; no matter how long search is continued (provided, of course, that the basic assumptions continue valid), detection probability cannot exceed the value P^*, where

$$P^* = 1-\exp\,(-S/\pi RU). \qquad (4'')$$

Second, note that most of the ultimate coverage is achieved early in the search. *Half* of the ultimate coverage $S/\pi RU$ (and more than half of the ultimate detection probability) is achieved in a search time T equal to R/U, the effective head start. Thus, search durations much in excess of $T=H$ are inherently inefficient; simultaneous search effort of several search vehicles at the outset is normally much more productive than protracted consecutive search by the same vehicles. Equation (3') indicates the improved coverage of simultaneous search; two similar vehicles searching simultaneously from the outset, for a search time $T = R/U$, the head start, give twice the coverage of a single vehicle. However, the two vehicles searching consecutively, each for a search time R/U, give only one third more coverage [$T/(T+H)$ increasing from 1/2 to 2/3 as duration is doubled] than that afforded by the initial searcher. This is in effect an argument for prompt saturation search about the point of fix, where feasible and warranted by the importance of the target.

ACKNOWLEDGEMENT

The author is warmly grateful for the interest and encouragement of Dr. L. R. Hafstad, Mr. R. M. Chapman, Mr. A. M. Bottoms, and Mr. L. H. Strauss. Valuable comments and suggested extensions and clarifications were received from Mr. Bottoms and Mr. Chapman, and have been largely incorporated in the text. In the processing of the slide rule, the author was much assisted by Mr. L. W. Barber and Mr. R. M. Chapman. The diligent and painstaking draftsmanship of Mr. P. A. Sweeney is also gratefully acknowledged. His excellent services greatly facilitated the preparation of a smooth prototype of the circular slide rule. The manuscript was typed by Mrs. Barbara Minars, whose careful work is much appreciated.

ADDENDUM SHEET

(For inclusion in report NRC:CUW:0374 "Detection Probability Computations for Random Search of an Expanding Area" July 1971)

FIXED AREA RANDOM SEARCH COMPUTATIONS

The accompanying calculator was designed to facilitate detection probability computations for search of an area which undergoes significant expansion during the conduct of the search. The calculator can, however, be readily used to compute detection probability for random search of an area of *fixed* size, according to the classical formula

$$P(T) = 1\text{-}\exp(-ST/A). \qquad \text{(A-1)}$$

Where:

S is search rate (area effectively swept out per unit time)

T is search duration

A is the size of the area know to contain the target

$P(T)$ is detection probability for search duration T

In the event that the area to be searched is circular, formula (A-1) may be written:

$$P(T) = 1\text{-}\exp(-ST/\pi R^2) \qquad \text{(A-2)}$$

where $\pi R^2 = A$. Regardless of the actual shape of the area, $R = \sqrt{A/\pi}$ may be regarded as an effective localization radius in the fixed-area situation and Side I of the calculator may be used to compute

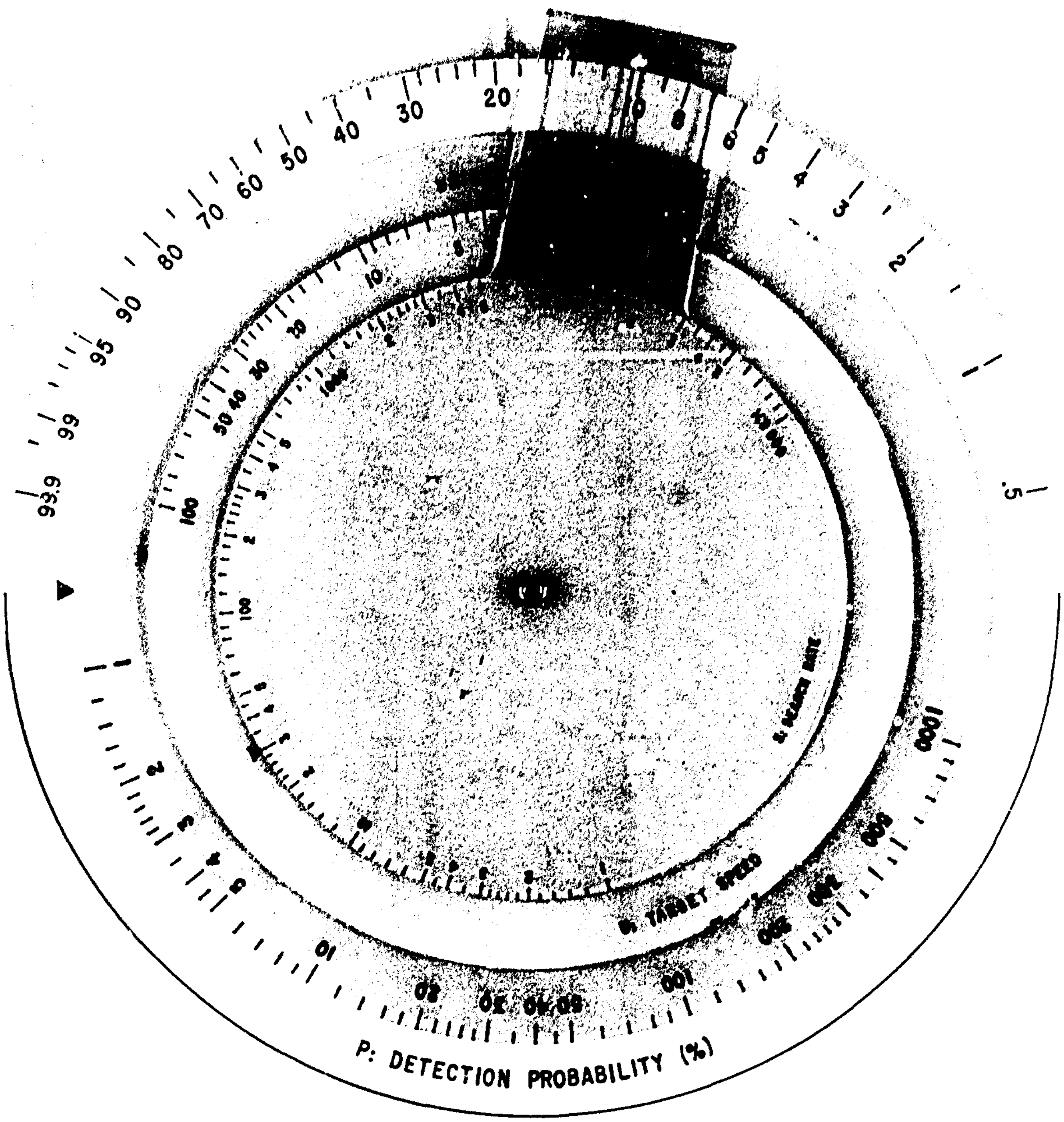
$$P(1) = 1\text{-}\exp(-S/\pi R^2) \qquad \text{(A-3)}$$

where $P(1)$ is the detection probability for a search of unit duration. Simply follow Side I instructions using the value R on both the R-dial and the U-dial (or use any pair of values R and U such that $\pi UR = A$).

Now, having computed the detection probability $P(1)$ for search of unit duration, turn to Side II and hold the index arm at the value $P(1)$ on the P-dial. Rotate the T-dial so that the value 1 (*not* the index arrow) appears under the hairline. Detection probability $P(T)$ may now be read by holding the T-dial fixed and rotating the index arm until the hairline appears over the value T on the T-dial. No use of the $H + T$-dial is made in this computation.

EXAMPLE

Suppose a fixed area of $A = 3000$ sq.mi. is to be searched at search rate $S = 200$ sq.mi. per hour. The effective localization radius R (such that $\pi R^2 = A$) is about 31 miles. Using $R = U = 31$ and $S = 200$ on Side I, we obtain $P(1) = 1\text{-}\exp(-\pi UR) = 6.4\%$. Turning now to Side II, and holding the index arm over 6.4% on the P-dial, rotate the T-dial until 1 appears under the hairline. Holding the T-dial fixed, rotate the index arm until the hairline appears over the desired value of search duration on the T-dial. For example, for $T = 6$ hours, $P(6) = 33\%$, and for $T = 15$ hours, $P(15) = 63\%$.



SIDE I: DETECTION PROBABILITY, PROLONGED SEARCH

INSTRUCTIONS

1. R is the estimated localization radius at the time search commences. SET THE VALUE OF R NEXT TO INDEX ▼ ON P-DIAL.
2. U is the estimated target speed. SET THE VALUE OF U NEXT TO INDEX ▼ ON R-DIAL.
3. S is the searcher's average search rate. If two or more searchers operate simultaneously, S is their combined search rate. HOLDING R AND U DIALS FIXED, MOVE INDEX ARM SO AS TO SET THE VALUE OF S NEXT TO INDEX ▼ ON U-DIAL.
4. P is the resulting detection probability for an indefinitely prolonged search at search rate S. READ P ON OUTER DIAL UNDER INDEX ARM HAIR-LINE.

NOTE

Any convenient units may be used for R, U, and S, but the units must be consistent throughout. Thus, if R is in nautical miles and U is in knots, S must be given in square nautical miles per hour.

EXAMPLE

If R = 30 nm, U = 15 knots, and S = 1200 nm^2 per hour, the resulting P is 1 - EXP ($-S/\pi UR$) = 57%. Set 30 on R-dial next to ▼ on P-dial, set 15 on U-dial next to ▼ on R-dial, set 1200 on S-dial next to ▼ on U-dial and read P on outer dial under hair-line.

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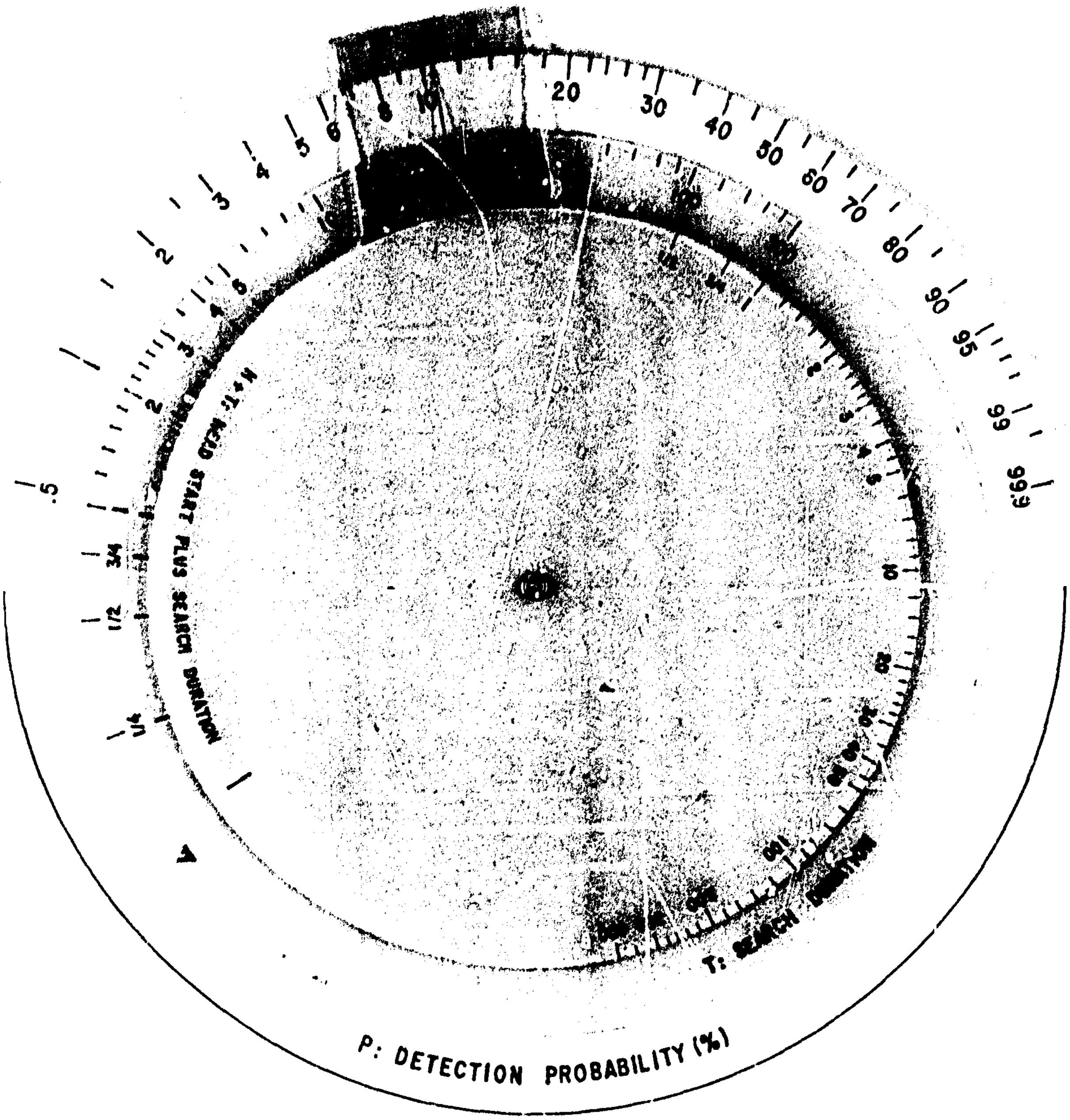
Slide Chart by Perrygraf Div. Nashua Corp., L.A., Cal. 90064 Printed in U.S.A.



SIDE II: DETECTION PROBABILITY, LIMITED SEARCH

INSTRUCTIONS

1. KEEP HAIR-LINE SET ON VALUE OF P OBTAINED FROM SIDE I COMPUTATION.
2. SET INDEX ▲ ON T-DIAL UNDER HAIR-LINE.
3. T is the duration of the limited search for which the detection probability is to be computed, and "Head Start" H equals the value of localization radius R used in the Side I computation divided by the target speed U used in the Side I computation: ADD H TO PLANNED SEARCH DURATION T, AND HOLD T-DIAL FIXED WHILE MOVING INDEX ARM SO THAT THE VALUE OF H + T ON THE H + T DIAL LIES DIRECTLY ADJACENT TO THE VALUE OF PLANNED SEARCH DURATION T ON THE T-DIAL.
4. P is the resulting detection probability for a *limited search* of duration T. READ P ON OUTER DIAL UNDER INDEX ARM HAIR-LINE.

NOTE

The same time units should be used for T and H as were used in the Side I computation. Thus, if U was in knots and S in nm^2 per hour, H and T should be in hours.

EXAMPLE

To compute the detection probability for a limited search of duration T = 3 hours, and with R = 30 nm, U = 15 knots, and S = 1200 nm^2, as before, proceed as follows: Using the P = 57% value obtained in the side I example, set the T dial index ▲ under the index arm hair-line aligned with 57%. The head start H from the side I computation is 2 hours (R = 30 nm divided by U = 15 knots). Holding the T-dial fixed so that index ▲ remains aligned with 57%, move index arm counterclockwise so as to place H + T = 5 directly adjacent to T = 3 on their respective dials. Now read detection probability P for 3 hour search duration under index arm hair-line:

$$P = 1\text{-}EXP\left\{(-S/\pi UR)(T/(H+T))\right\} = 40\%$$